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Minimum Mass Structures
with Specified Natural Frequencies

by

A. Miele

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Minimum Mass Structures with Specified Natural Frequencies[1,2]

by

A. Miele[3]

Abstract. The problem of the axial vibration of a cantilever beam is investigated numerically. The mass distribution that minimizes the total mass for a given fundamental frequency constraint is determined using both the sequential ordinary gradient-restoration algorithm (SOGRA) and an ad hoc modification of the modified quasilinearization algorithm (MQA).

Key Words. Structural optimization, cantilever beams, axial vibrations, fundamental frequency constraint, numerical methods, sequential ordinary gradient-restoration algorithm, modified quasilinerization algorithm.

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[2]The author is indebted to Dr. V.B. Venkayya, Wright-Patterson AFB, Ohio, for suggesting the topic. He is also indebted to Messieurs B.P. Mohanty and A.K. Wu for computational assistance.

[3]Professor of Astronautics and Mathematical Sciences, Rice University, Houston, Texas.

Notation

E	Modulus of elasticity, lb ft^{-2}
I	Normalized mass of the beam, $I = M_*/M_o$
L	Length of the beam, ft
m	Normalized mass per unit length, $m = ML/M_o$
M	Mass per unit length, lb ft^{-2} sec^2
M_o	Tip mass, lb ft^{-1} sec^2
M_*	Total mass of the beam, lb ft^{-1} sec^2
x	Normalized axial coordiante, $x = X/L$
X	Axial coordinate, ft
u	Normalized axial displacement, $u = Y(X)/Y(L)$
Y	Axial displacement, ft
β	Frequency parameter, $\beta = \omega L\sqrt{(\rho/E)}$
ρ	Density, lb ft^{-4} sec^2
ω	Natural frequency, sec^{-1}

Superscript

'	Derivative with respect to the normalized axial coordinate x (for example, $u' = du/dx$)

1. Introduction

In this report, we consider the numerical determination of minimum mass structures with specified natural frequencies. Specifically, we investigate the problem of the axial vibration of a cantilever beam: we determine the mass distribution that minimizes the total mass for a given fundamental frequency constraint.

The above problem has been investigated analytically by Turner (Ref. 1) within the frame of the following formulation: minimize the integral

$$I = \int_0^1 m dx \, , \tag{1}$$

with respect to the functions $u(x)$ and $m(x)$ which satisfy the differential equation

$$(mu')' + \beta^2 mu = 0 \tag{2}$$

and the boundary conditions[4]

$$u(0) = 0 \, , \tag{3}$$

$$u(1) = 1 \, , \qquad m(1)u'(1) = \beta^2 \, . \tag{4}$$

In the above equations, m is the mass per unit length, u is the axial displacement, β is the frequency parameter, and I is the total mass of the rod. The prime denotes derivative

[4]Equation (4-1) is a normalization condition for the displacement function $u(x)$.

with respect to the axial coordinate x.

Turner (Ref. 1) proved that the above problem admits the following analytical solution:

$$u = \sinh(\beta x)/\sinh(\beta), \tag{5}$$

$$m = \beta\sinh(\beta)\cosh(\beta)/\cosh^2(\beta x)\ , \tag{6}$$

with the implication that the minimum value of the total mass of the rod is

$$I = \sinh^2(\beta)\ . \tag{7}$$

2. Optimal Control Formulation

Prior to investigating numerically the above problem, we reformulate it by using the terminology of optimal control theory.

First Formulation. We introduce the following variables:

$$t = x , \qquad x_1 = u , \qquad x_2 = u' , \qquad w = u'' , \qquad x_3 = m , \tag{8}$$

and rewrite problem (1)-(4) as follows: minimize the integral

$$I = \int_0^1 x_3 dt , \tag{9}$$

with respect to the functions $x_1(t)$, $x_2(t)$, $x_3(t)$, $w(t)$ which satisfy the differential constraints

$$\dot{x}_1 = x_2 , \tag{10}$$

$$\dot{x}_2 = w , \tag{11}$$

$$\dot{x}_3 = -(w + \beta^2 x_1) x_3 / x_2 , \tag{12}$$

and the boundary conditions

$$x_1(0) = 0 , \tag{13}$$

$$x_1(1) = 1 , \qquad x_2(1) x_3(1) = \beta^2 . \tag{14}$$

In the above equations, $x_1(t)$, $x_2(t)$, $x_3(t)$ denote the state variables and $w(t)$ denotes the control variable.

<u>Second Formulation</u>. Upon introducing the auxiliary state variable $x_4(t)$ defined by

$$x_4 = x_2 x_3 \tag{15}$$

and upon eliminating $x_3(t)$, we see that the previous problem can be reformulated as follows: minimize the integral

$$I = \int_0^1 (x_4/x_2)\,dt\ , \tag{16}$$

with respect to the functions $x_1(t)$, $x_2(t)$, $x_4(t)$, $w(t)$ which satisfy the differential constraints

$$\dot{x}_1 = x_2\ , \tag{17}$$

$$\dot{x}_2 = w\ , \tag{18}$$

$$\dot{x}_4 = -\beta^2 x_1 x_4/x_2\ , \tag{19}$$

and the boundary conditions

$$x_1(0) = 0\ , \tag{20}$$

$$x_1(1) = 1\ , \qquad x_4(1) = \beta^2\ . \tag{21}$$

Once a solution $x_1(t)$, $x_2(t)$, $x_4(t)$, $w(t)$ is obtained, the eliminated state variable $x_3(t)$ can be computed from

$$x_3 = x_4/x_2\ . \tag{22}$$

Redundant Constraints. Problem (16)-(21) is singular, in that the control $w(t)$ appears linearly. Ordinarily, meaningful solutions can only be obtained by imposing some appropriate bound on the state and/or the control, for instance, a bound of the form

$$0 \leq w(t) \leq k \,. \tag{23}$$

If a bound of type (23) is imposed, then the optimal solution is generally composed of subarcs of the following kind:

$$\text{(i)} \quad w = 0 \,, \tag{24-1}$$

$$\text{(ii)} \quad H_w = 0 \,, \tag{24-2}$$

$$\text{(iii)} \quad w = k \,, \tag{24-3}$$

where H denotes the Hamiltonian function

$$H = x_4/x_2 - \lambda_1 x_2 - \lambda_2 w + \lambda_4 \beta^2 x_1 x_4/x_2 \,. \tag{25}$$

Since

$$H_w = -\lambda_2 \,, \tag{26}$$

we see that Eqs. (24) can be rewritten as

$$\text{(i)} \quad w = 0 \,, \tag{27-1}$$

$$\text{(ii)} \quad \lambda_2 = 0 \,, \tag{27-2}$$

$$\text{(iii)} \quad w = k \,. \tag{27-3}$$

For the particular problem under consideration, certain simplifying circumstances exist. Owing to the fact that

$$x_2(0) = \text{free}\,, \qquad x_2(1) = \text{free}\,, \tag{28}$$

the natural boundary conditions arising from the transversality condition require that

$$\lambda_2(0) = 0\,, \qquad \lambda_2(1) = 0\,. \tag{29}$$

Since these boundary conditions are consistent with (27-2), it is natural to postulate that the extremal arc includes a single subarc (ii), along which

$$\lambda_2(t) = 0\,, \qquad 0 \le t \le 1\,. \tag{30}$$

The result represented by (30) has two implications. First, the two-sided inequality (23) is redundant, provided k is sufficiently large. Second, the differential constraint (18) is also redundant. If one disregards (18), then the variable $x_2(t)$ appears in nonderivated form only; hence, $x_2(t)$ becomes the control variable $v(t)$ of the following simplified formulation.

<u>Third Formulation</u>. Upon disregarding (18) and (23), and upon setting

$$v = x_2\,, \tag{31}$$

we obtain the following optimal control problem: minimize the integral

$$I = \int_0^1 (x_4/v)\,dt\,, \tag{32}$$

with respect to the functions $x_1(t)$, $x_4(t)$, $v(t)$ which satisfy the differential constraints

$$\dot{x}_1 = v\,, \tag{33}$$

$$\dot{x}_4 = -\beta^2 x_1 x_4/v\,, \tag{34}$$

and the boundary conditions

$$x_1(0) = 0\,, \tag{35}$$

$$x_1(1) = 1\,, \qquad x_4(1) = \beta^2\,. \tag{36}$$

Once a solution $x_1(t)$, $x_4(t)$, $v(t)$ is obtained, the eliminated variables $x_2(t)$ and $x_3(t)$ can be computed from

$$x_2 = v\,, \tag{37}$$

$$x_3 = x_4/v\,. \tag{38}$$

This formulation differs from the previous two formulations for two reasons: (i) it involves two differential constraints, rather than three; and (ii) the resulting optimal control problem is nonsingular (Refs. 2-3).

3. Analytical Solution

It can be readily verified that the previous optimal control problem has the following analytical solution:

$$x_1 = \sinh(\beta t)/\sinh(\beta) , \tag{39-1}$$

$$x_4 = \beta^2 \cosh(\beta)/\cosh(\beta t) , \tag{39-2}$$

$$v = \beta \cosh(\beta t)/\sinh(\beta) , \tag{39-3}$$

so that

$$x_2 = \beta \cosh(\beta t)/\sinh(\beta) , \tag{40-1}$$

$$x_3 = \beta \sinh(\beta) \cosh(\beta)/\cosh^2(\beta t) , \tag{40-2}$$

and

$$I = \sinh^2(\beta) . \tag{41}$$

4. Numerical Solutions

Problem (32)-(36) was solved numerically at Rice University, Houston, Texas, using both the sequential ordinary gradient-restoration algorithm (SOGRA, Ref. 4) and an ad hoc variation of the modified quasilinearization algorithm (MQA, Ref. 5). Both algorithms were programmed in FORTRAN IV. Computations were performed using an IBM 370/155 computer and double precision arithmetic. The interval of integration was divided into 100 steps. The differential systems were integrated using Hamming's modified predictor-corrector method with a special Runge-Kutta procedure to start the integration routine. Definite integrals (such as I, P, Q) were computed using a modified Simpson's rule. Here, I denotes the functional being minimized, P is the cumulative constraint error, and Q is the cumulative error in the optimality conditions. For the definition of P and Q, see Refs. 4 and 5.

Experimental Conditions, SOGRA. The following nominal funtions were employed in order to start the sequential gradient-restoration algorithm:

$$x_1(t) = t\ , \tag{42-1}$$

$$x_4(t) = \beta^2 \exp[\beta^2(1-t^2)/2]\ , \tag{42-2}$$

$$v(t) = 1\ . \tag{42-3}$$

These nominal functions are consistent with the constraints (33)-(36). Three values of the frequency parameter were employed, namely,

$$\beta = \pi/4 , \qquad \beta = \pi/2 , \qquad \beta = \pi . \tag{43}$$

The sequential ordinary gradient-restoration algorithm was programmed to stop whenever a solution consistent with the following inequalities was obtained:[5]

$$P \leq E\text{-}08 , \qquad Q \leq E\text{-}03 . \tag{44}$$

Convergence was achieved in $N = 2$ iterations for $\beta = \pi/4$, $N = 9$ iterations for $\beta = \pi/2$, and $N = 29$ iterations for $\beta = \pi$.

Experimental Conditions, MQA. The converged solutions generated with SOGRA were employed as the nominal functions for the modified quasilinearization algorithm. The modified quasilinearization algorithm was programmed to stop whenever a solution consistent with the following inequalities was obtained:

$$P \leq E\text{-}12 , \qquad Q \leq E\text{-}12 . \tag{45}$$

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[5] The symbol $E \pm ab$ stands for $10^{\pm ab}$.

Convergence was achieved in $N = 1$ iteration for $\beta = \pi/4$, $N = 1$ iteration for $\beta = \pi/2$, and $N = 4$ iterations for $\beta = \pi$.

Numerical Results. Table 1 presents summary results pertaining to SOGRA at convergence. For each value of the frequency parameter β, the table shows the number of iterations for convergence N, the computed value of the functional I, the exact value of the functional I_e, the number of correct significant digits M (determined by comparing I with I_e), the constraint error P, and the error in the optimality conditions Q. Clearly, as far as the minimum mass is concerned, the solutions obtained are precise to $M = 4$ significant digits for $\beta = \pi/4$, to $M = 5$ significant digits for $\beta = \pi/2$, and to $M = 2$ significant digits for $\beta = \pi$.

Table 2 presents summary results pertaining to MQA at convergence. Clearly, as far as the minimum mass is concerned, the solutions obtained are precise to $M = 7$ significant digits for all values of the frequency parameter β.

Tables 3-5 present the converged solutions $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, $v(t)$ obtained with SOGRA, and Tables 6-8 present the converged solutions obtained with MQA. The latter solutions agree with the analytical solutions (39)-(40) to at least 4 significant digits at each time station t.

5. Discussion and Conclusions

In this report, we have investigated numerically the problem of the axial vibration of a cantilever beam. We have determined the mass distribution that minimizes the total mass for a given fundamental frequency constraint. The original problem is difficult because of its singular character (see first and second formulations). However, the transformed problem is nonsingular (one of the differential constraints is redundant, see third formulation).

We solved the above problem employing both the sequential ordinary gradient-restoration algorithm (Ref. 4) and the modified quasilinearization algorithm (Ref. 5). Using SOGRA, we determined a preliminary solution relatively close to the optimum solution. Then, this preliminary solution was employed in conjunction with MQA in order to determine a more precise approximation to the optimum solution.

Concerning the minimum value of the functional I, the SOGRA solutions are precise to $M = 4$ significant digits for $\beta = \pi/4$, $M = 5$ significant digits for $\beta = \pi/2$, and $M = 2$ significant digits for $\beta = \pi$. On the other hand, the MQA solutions are precise to $M = 7$ significant digits for all values of the frequency parameter β.

Table 1. Summary results, SOGRA.

β	N	I	I_e	M	P	Q
$\pi/4$	2	0.7545936E+00	0.7545892E+00	4	0.21E-32	0.15E-04
$\pi/2$	9	0.5295951E+01	0.5295977E+01	5	0.99E-08	0.88E-03
π	29	0.1346253E+03	0.1333734E+03	2	0.60E-33	0.97E-03

Table 2. Summary results, MQA.

β	N	I	I_e	M	P	Q
$\pi/4$	1	0.7545892E+00	0.7545892E+00	7	0.23E-14	0.10E-30
$\pi/2$	1	0.5295977E+01	0.5295977E+01	7	0.74E-17	0.10E-30
π	4	0.1333734E+03	0.1333734E+03	7	0.34E-13	0.10E-30

Table 3. SOGRA solution, $\beta = \pi/4$, $N = 2$, $P \le E\text{-}08$, $Q \le E\text{-}03$.

t	x_1	x_2	x_3	x_4	v
0.0	0.0000E+00	0.9011E+00	0.9064E+00	0.8168E+00	0.9011E+00
0.1	0.9021E-01	0.9042E+00	0.9005E+00	0.8143E+00	0.9042E+00
0.2	0.1810E+00	0.9133E+00	0.8833E+00	0.8068E+00	0.9133E+00
0.3	0.2730E+00	0.9285E+00	0.8558E+00	0.7946E+00	0.9285E+00
0.4	0.3669E+00	0.9496E+00	0.8194E+00	0.7781E+00	0.9496E+00
0.5	0.4632E+00	0.9765E+00	0.7759E+00	0.7577E+00	0.9765E+00
0.6	0.5624E+00	0.1009E+01	0.7273E+00	0.7339E+00	0.1009E+01
0.7	0.6652E+00	0.1047E+01	0.6755E+00	0.7074E+00	0.1047E+01
0.8	0.7720E+00	0.1090E+01	0.6223E+00	0.6787E+00	0.1090E+01
0.9	0.8835E+00	0.1138E+01	0.5693E+00	0.6483E+00	0.1138E+01
1.0	0.1000E+01	0.1191E+01	0.5175E+00	0.6168E+00	0.1191E+01

I = 0.7545936E+00

Table 4. SOGRA solution, $\beta = \pi/2$, $N = 9$, $P \leq E-08$, $Q \leq E-03$.

t	x_1	x_2	x_3	x_4	v
0.0	0.0000E+00	0.6828E+00	0.9060E+01	0.6186E+01	0.6828E+00
0.1	0.6854E-01	0.6907E+00	0.8847E+01	0.6111E+01	0.6907E+00
0.2	0.1387E+00	0.7153E+00	0.8238E+01	0.5893E+01	0.7153E+00
0.3	0.2122E+00	0.7583E+00	0.7328E+01	0.5557E+01	0.7583E+00
0.4	0.2910E+00	0.8205E+00	0.6261E+01	0.5137E+01	0.8205E+00
0.5	0.3770E+00	0.9025E+00	0.5173E+01	0.4669E+01	0.9025E+00
0.6	0.4722E+00	0.1007E+01	0.4154E+01	0.4185E+01	0.1007E+01
0.7	0.5794E+00	0.1139E+01	0.3254E+01	0.3708E+01	0.1139E+01
0.8	0.7011E+00	0.1301E+01	0.2504E+01	0.3258E+01	0.1301E+01
0.9	0.8405E+00	0.1490E+01	0.1908E+01	0.2844E+01	0.1490E+01
1.0	0.1000E+01	0.1701E+01	0.1449E+01	0.2467E+01	0.1701E+01

$I = 0.5295951E+01$

Table 5. SOGRA solution, $\beta = \pi$, $N = 29$, $P \leq$ E-08, $Q \leq$ E-03 .

t	x_1	x_2	x_3	x_4	v
0.0	0.0000E+00	0.2857E+00	0.4264E+03	0.1218E+03	0.2857E+00
0.1	0.2899E-01	0.3100E+00	0.3747E+03	0.1161E+03	0.3100E+00
0.2	0.6136E-01	0.3466E+00	0.2925E+03	0.1014E+03	0.3466E+00
0.3	0.9965E-01	0.4254E+00	0.1941E+03	0.8261E+02	0.4254E+00
0.4	0.1478E+00	0.5471E+00	0.1175E+03	0.6431E+02	0.5471E+00
0.5	0.2109E+00	0.7252E+00	0.6715E+02	0.4870E+02	0.7252E+00
0.6	0.2958E+00	0.9891E+00	0.3677E+02	0.3637E+02	0.9891E+00
0.7	0.4118E+00	0.1344E+01	0.2008E+02	0.2699E+02	0.1344E+01
0.8	0.5663E+00	0.1748E+01	0.1132E+02	0.1979E+02	0.1748E+01
0.9	0.7621E+00	0.2168E+01	0.6547E+01	0.1419E+02	0.2168E+01
1.0	0.1000E+01	0.2588E+01	0.3813E+01	0.9869E+01	0.2588E+01

I = 0.1346253E+03

Table 6. MQA solution, $\beta = \pi/4$, $N = 1$, $P \leq E-12$, $Q \leq E-12$.

t	x_1	x_2	x_3	x_4	v
0.0	0.0000E+00	0.9041E+00	0.9037E+00	0.8170E+00	0.9041E+00
0.1	0.9050E-01	0.9069E+00	0.8981E+00	0.8145E+00	0.9069E+00
0.2	0.1815E+00	0.9153E+00	0.8817E+00	0.8071E+00	0.9153E+00
0.3	0.2737E+00	0.9293E+00	0.8553E+00	0.7949E+00	0.9293E+00
0.4	0.3676E+00	0.9491E+00	0.8200E+00	0.7783E+00	0.9491E+00
0.5	0.4637E+00	0.9747E+00	0.7775E+00	0.7578E+00	0.9747E+00
0.6	0.5627E+00	0.1006E+01	0.7293E+00	0.7340E+00	0.1006E+01
0.7	0.6652E+00	0.1044E+01	0.6774E+00	0.7074E+00	0.1044E+01
0.8	0.7718E+00	0.1088E+01	0.6234E+00	0.6786E+00	0.1088E+00
0.9	0.8831E+00	0.1139E+01	0.5688E+00	0.6482E+00	0.1139E+01
1.0	0.1000E+01	0.1197E+01	0.5150E+00	0.6168E+00	0.1197E+01

$I = 0.7545892E+00$

Table 7. MQA solution, $\beta = \pi/2$, $N = 1$, $P \leq E-12$, $Q \leq E-12$.

t	x_1	x_2	x_3	x_4	v
0.0	0.0000E+00	0.6825E+00	0.9070E+01	0.6191E+01	0.6825E+00
0.1	0.6853E-01	0.6910E+00	0.8850E+01	0.6115E+01	0.6910E+00
0.2	0.1387E+00	0.7165E+00	0.8230E+01	0.5897E+01	0.7165E+00
0.3	0.2124E+00	0.7597E+00	0.7320E+01	0.5562E+01	0.7597E+00
0.4	0.2913E+00	0.8217E+00	0.6257E+01	0.5142E+01	0.8217E+00
0.5	0.3774E+00	0.9041E+00	0.5169E+01	0.4673E+01	0.9041E+00
0.6	0.4729E+00	0.1008E+01	0.4152E+01	0.4188E+01	0.1008E+01
0.7	0.5800E+00	0.1138E+01	0.3260E+01	0.3711E+01	0.1138E+01
0.8	0.7015E+00	0.1296E+01	0.2514E+01	0.3260E+01	0.1296E+01
0.9	0.8403E+00	0.1486E+01	0.1913E+01	0.2843E+01	0.1486E+01
1.0	0.1000E+01	0.1712E+01	0.1440E+01	0.2467E+01	0.1712E+01

I = 0.5295977E+01

Table 8. MQA solution, $\beta = \pi$, $N = 4$, $P \leq E-12$, $Q \leq E-12$.

t	x_1	x_2	x_3	x_4	v
0.0	0.0000E+00	0.2720E+00	0.4205E+03	0.1144E+03	0.2720E+00
0.1	0.2765E-01	0.2855E+00	0.3816E+03	0.1089E+03	0.2855E+00
0.2	0.5805E-01	0.3275E+00	0.2901E+03	0.9502E+02	0.3275E+00
0.3	0.9423E-01	0.4020E+00	0.1925E+03	0.7740E+02	0.4020E+00
0.4	0.1397E+00	0.5166E+00	0.1166E+03	0.6024E+02	0.5166E+00
0.5	0.1992E+00	0.6825E+00	0.6680E+02	0.4559E+02	0.6825E+00
0.6	0.2785E+00	0.9164E+00	0.3705E+02	0.3395E+02	0.9164E+00
0.7	0.3855E+00	0.1241E+01	0.2019E+02	0.2506E+02	0.1241E+01
0.8	0.5309E+00	0.1690E+01	0.1089E+02	0.1841E+02	0.1690E+01
0.9	0.7292E+00	0.2306E+01	0.5847E+01	0.1349E+02	0.2306E+01
1.0	0.1000E+01	0.3153E+01	0.3129E+01	0.9869E+01	0.3153E+01

$I = 0.1333734E+03$

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The problem of the axial vibration of a cantilever beam is investigated numerically. The mass distribution that minimizes the total mass for a given fundamental frequency constraint is determined using both the sequential ordinary gradient- (OVER)

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restoration algorithm (SOGRA) and an ad hoc modification of the modified quasilinearization algorithm (MQA).